# NATIONAL SENIOR CERTIFICATE 

## GRADE 12

JUNE 2022

## MATHEMATICS P1

MARKS: 150

TIME: 3 hours

This question paper consists of 12 pages, including an information sheet.

## INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions. Answer ALL the questions.
2. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
3. Answers only will not necessarily be awarded full marks.
4. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
5. If necessary, round off answers to TWO decimal places, unless stated otherwise.
6. Diagrams are NOT necessarily drawn to scale.
7. An information sheet, with formulae, is included at the end of the question paper.
8. Number the answers correctly according to the numbering system used in this question paper.
9. Write neatly and legibly.

## QUESTION 1

1.1 Solve for $x$, in each of the following:
1.1.1 $\quad x^{2}=-4 x$
1.1.2 $x^{2}+x-1=0 \quad$ (correct to TWO decimal places)
1.1.3 $\sqrt{x+4}-\frac{4}{\sqrt{x-2}}=0$
1.1.4 $(x+2)(-3 x+1)>0$
1.2 Solve for $x$ and $y$ simultaneously:
$3-y+2 x=0$
$6 x+4 y^{2}=3+5 x y$
1.3 Given that $9 x^{2}-12 p x=-4 p^{2}$. For which value(s) of $p$ will the equation have equal roots?

## QUESTION 2

2.1 Given the geometric sequence: $\frac{9}{2} ; 9 ; 18 ; \ldots ; 2304$
2.1.1 Determine the value of $r$, the common ratio.
2.1.2 How many terms are there in the sequence?
2.2 Given: $\sum_{k=1}^{\infty} 6(m)^{k-1}=12$. Determine the value of $m$.
2.3 The $3^{\text {rd }}$ term of a geometric series is 18 and the $5^{\text {th }}$ term is 162 . Determine the sum of the first 7 terms, where $r<0$.
2.4 The general term of a quadratic number pattern is $T_{n}=a n^{2}+b n+c$ and its first term is 8 . The general term of the first differences of the pattern is $t_{k}=4 k-2$.
2.4.1 Determine the next two terms of the quadratic number pattern, $T_{n}$.
2.4.2 Hence, or otherwise, show that the general term of the quadratic number pattern is given by $T_{n}=2 n^{2}-4 n+10$.
2.4.3 Which term of the quadratic number pattern will be equal to 3050 ?

## QUESTION 3

The following figure represents a pattern of shaded triangles placed on a white rectangular board. The triangles all have equal bases of 4 units in length. The height of the first triangle is 1 unit. Each triangle's height thereafter is 1 unit more than the previous one.

3.1 Determine the area of the first triangle.
3.2 Determine the area of the $26^{\text {th }}$ triangle.
3.3 The triangles are placed on a rectangular board, with length 104 units, as shown above. Determine the area of the unshaded part of the white rectangular board, that is, the area of the part not covered by the shaded triangles.

## QUESTION 4

Given: $f(x)=\frac{8}{x-2}+2$
4.1 Write down the domain of $f$.
4.2 Calculate the $y$-intercept of $f$.
4.3 Calculate the $x$-intercept of $f$.
4.4 Sketch the graph of $f$, clearly indicating the coordinates of the $x$ and $y$-intercepts as well as the asymptotes.
4.5 If $y=-x+k$ is an equation of the line of symmetry of $f$, determine the value of $k$.
4.6 Determine the equation of the graph formed if $f$ is shifted 3 units to the right and then reflected across the $x$-axis.

## QUESTION 5

The graphs of $f(x)=2(x+1)^{2}-8$ and $g(x)=\left(\frac{1}{2}\right)^{x}$ are represented in the sketch below. P and Q are the $x$-intercepts of $f$ and R is the turning point of $f$. $\mathrm{A}(-2 ; 4)$ is a point on the graph of $g$.

5.1 Write down the equation of the axis of symmetry of $f$.
5.2 Write down the coordinates of R , the turning point of $f$.
5.3 Determine the coordinates of P and Q .
5.4 Determine the equation of $g^{-1}$, the inverse of $g$, in the form $y=\ldots$
5.5 Sketch the graph of $g^{-1}$ in your ANSWER BOOK. Clearly indicate the intercept with the axis and at least ONE other point on $g^{-1}$.
5.6 For which value(s) of $x$, is:
5.6.1 $\quad g^{-1}(x) \geq-2 ?$
5.6.2 $\quad$. $f(x)<0$ ?

## QUESTION 6

6.1 How long must R50 000 be invested, in order for it to double at an interest rate of $8,5 \%$ p.a. on the straight-line method? (Give your answer in years and months.)
6.2 A cellphone valued at R24 000 depreciates at $18 \%$ p.a. on the reducing balance method. Determine the value of the cellphone after 3 years.
6.3 $\quad \mathrm{R} x$ ( $x$ Rand) is invested into an account at an interest rate of $12 \%$ p.a. compounded monthly. Three years later R2x ( $2 x$ Rand) is deposited into the same account. After 7 years there is R276558,75 in the account. Determine how much money was invested at the beginning.
(that is, the value of $x$ )

## QUESTION 7

7.1 Determine $f^{\prime}(x)$ from first principles if $f(x)=-2 x^{2}+x$.
7.2 Determine:
7.2.1 $\quad D_{x}\left[\frac{-5 x}{\sqrt{x}}-\frac{x^{2}}{5}\right]$
7.2.2 $\frac{d}{d x}\left[\left(x+\frac{2}{x}\right)\left(x-\frac{2}{x}\right)\right]$

## QUESTION 8

The sketch below represents the functions $f(x)=x^{3}+b x^{2}+c x+d$ and $g(x)=a x+q$.
The points $\mathrm{A}, \mathrm{B}(2 ;-16)$ and C are the points where the two graphs intersect.
$\mathrm{C}(6 ; 0)$ is a $x$-intercept of $f$, while L and M are the turning points of $f$.

8.1 Show that $b=-5, c=-8$ and $d=12$ if it is given that, $f^{\prime}(x)=3 x^{2}-10 x-8$.
8.2 Determine the coordinates of the turning points, L and M , of $f$.
8.3 Determine the equation of $g$.
8.4 If it is further given that the coordinates of point A are $(x ;-36)$, determine the length of AM.
8.5 For which value(s) of $x$ :
8.5.1 is the graph, $f$ increasing?
8.5.2 is the graph, $f$ concave down?

## QUESTION 9

In the figure below, $\triangle \mathrm{ABE}$ has a base of length $x$ metres.
The base and the perpendicular height of the triangle add up to 10 metres.
The triangle is mounted on a rectangle BCDE which has a perimeter of 32 metres.

9.1 Show that the area of the figure ABCDE is equal to $-\frac{3}{2} x^{2}+21 x \mathrm{~m}^{2}$.
9.2 Determine the value of $x$ for which ABCDE has a maximum area.
9.3 Hence, determine the maximum area of ABCDE .

## QUESTION 10

10.1 In a survey, 1530 people were asked if they had ever broken a limb. The results of the survey were as follows:

|  | Broken a limb | Not Broken a limb | Total |
| :--- | :---: | :---: | :---: |
| Male | 463 | $b$ | 782 |
| Female | $a$ | $c$ | $d$ |
| Total | 913 | 617 | 1530 |

10.1.1 Calculate the values of $a, b, c$, and $d$.
10.1.2 If a person is chosen at random, what is the probability that it will be a female who has not broken a limb?
10.2 Two learners are selected at random from a group of 10 boys and 12 girls. Determine the probability that:
10.2.1 They are both girls

### 10.2.2 One is a boy and one is a girl

10.3 In the Venn diagram below, M and N are independent events.


Calculate, giving answers correct to two decimal places:
10.3.1 The value of $x$.
10.3.2 The value of $y$.

## INFORMATION SHEET: MATHEMATICS

$x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}$
$A=P(1+n i)$

$$
A=P(1-n i)
$$

$$
A=P(1-i)^{n}
$$

$$
A=P(1+i)^{n}
$$

$T_{n}=a+(n-1) d$
$S_{n}=\frac{n}{2}(2 a+(n-1) d)$
$T_{n}=a r^{n-1} \quad S_{n}=\frac{a\left(r^{n}-1\right)}{r-1} ; r \neq 1 \quad S_{\infty}=\frac{a}{1-r} ;-1<r<1$
$F=\frac{x\left[(1+i)^{n}-1\right]}{i}$ $P=\frac{x\left[1-(1+i)^{-n}\right]}{i}$
$f^{\prime}(x)=\lim _{h \rightarrow 0} \frac{f(x+h)-f(x)}{h}$
$d=\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}}$
$\mathrm{M}\left(\frac{x_{1}+x_{2}}{2} ; \frac{y_{1}+y_{2}}{2}\right)$
$y=m x+c \quad y-y_{1}=m\left(x-x_{1}\right) \quad m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \quad m=\tan \theta$
$(x-a)^{2}+(y-b)^{2}=r^{2}$
In $\triangle A B C$ :
$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C} \quad a^{2}=b^{2}+c^{2}-2 b c \cdot \cos A \quad$ area $\triangle A B C=\frac{1}{2} a b \cdot \sin C$
$\sin (\alpha+\beta)=\sin \alpha \cdot \cos \beta+\cos \alpha \cdot \sin \beta$
$\sin (\alpha-\beta)=\sin \alpha \cdot \cos \beta-\cos \alpha \cdot \sin \beta$
$\cos (\alpha+\beta)=\cos \alpha \cdot \cos \beta-\sin \alpha \cdot \sin \beta$ $\cos (\alpha-\beta)=\cos \alpha \cdot \cos \beta+\sin \alpha \cdot \sin \beta$
$\cos 2 \alpha=\left\{\begin{array}{l}\cos ^{2} \alpha-\sin ^{2} \alpha \\ 1-2 \sin ^{2} \alpha \\ 2 \cos ^{2} \alpha-1\end{array}\right.$ $\sin 2 \alpha=2 \sin \alpha \cdot \cos \alpha$

$$
\bar{x}=\frac{\sum x}{n}
$$

$$
\partial^{2}=\frac{\sum_{i=1}^{n}\left(x_{i}-\bar{x}\right)^{2}}{n}
$$

$$
P(A)=\frac{n(A)}{n(S)}
$$

$P(A$ or $B)=P(A)+P(B)-P(A$ and $B)$

$$
\hat{y}=a+b x \quad b=\frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^{2}}
$$

