

10.2 If it is further given that $CD = 2$ units and $DE = 6$ units, calculate the length of:

- 10.2.1 BC
10.2.2 DB

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10.2.2 DB

(4)

(17)

(150)

Independent Examination Board

November 2018

Paper 1

Marks: 150

Time: 3 hours

Onafhanklike Eksamenraad

November 2018

Vraestel 1

Punte: 150

Tyd: 3 uur

G1

SECTION A

AFDELING A

Question 1

Vraag 1

(a) The 100th term of an arithmetic sequence is 512 and its common difference is 7. Determine the first term of the sequence.

(a) Die 100^{ste} term van 'n rekenkundige ry is 512 en die gemene verskil van die ry is 7. Bepaal die eerste term van die ry.

(3)

(b) The general term of a sequence is $T_n = 2n + 3$.

(b) Die algemene term van 'n ry is $T_n = 2n + 3$.

(1) Show that the sequence is arithmetic.

(1) Toon dat die ry rekenkundig is.

(3)

(2) Determine, in terms of n , a simplified expression for S_n , the sum of the first n terms.

(2) Bepaal in terme van n 'n vereenvoudigde uitdrukking vir S_n , die som van die eerste n terme.

(3)

(c) Consider the given quadratic sequence: 4, 7, 14, 25, ... Determine a simplified expression for the n^{th} term of the sequence.

(c) Beskou die gegewe kwadraalse reë: 4, 7, 14, 25, ... Bepaal 'n vereenvoudigde uitdrukking vir die n^{de} term van die ry.

(4)

Question 2

Vraag 2

(a) Given: $\sum_{n=1}^x 108 \times \left(\frac{2}{3}\right)^n$

(a) Gegewe: $\sum_{n=1}^x 108 \times \left(\frac{2}{3}\right)^n$

(13)

(1) Determine the first two terms.

(1) Bepaal die eerste twee terme.

(2)

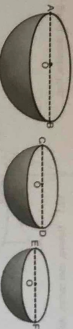
(2) If $\sum_{n=1}^x 108 \times \left(\frac{2}{3}\right)^n = \frac{520}{3}$, determine the value of x .

(2) Indien $\sum_{n=1}^x 108 \times \left(\frac{2}{3}\right)^n = \frac{520}{3}$, bepaal die waarde van x .

(4)

(b) Hollow plastic hemispheres are created such that each successive one fits into the previous one. The radii $OB = 21$ cm, $OD = 3$ cm and $OF = \frac{3}{7}$ cm.

(b) Hol plastiek-hemisfere word geskep sodanig dat elke opeenvolgende een in die vorige een pas. Die radiusse is $OB = 21$ cm, $OD = 3$ cm en $OF = \frac{3}{7}$ cm.



Determine the sum of the outer surface areas, as shaded in the diagram, of all such hemispheres created by continuing the pattern indefinitely.

Bepaal die som van die buiteoppervlakte, soos gearseer in die diagram, van al sodanige hemisfere wat geskep word deur die patroon onbepaald voor te sit.

Useful formula: Surface Area of a hemisphere = $2\pi r^2$.

Nuttige formule: Buiteoppervlakte van 'n hemisfeer = $2\pi r^2$.

(5)

Question 3

Vraag 3

(a) Given: $f(x) = x^2 - 3x - 4$ and $g(x) = x + 1$. Calculate the following:

(a) Gegewe: $f(x) = x^2 - 3x - 4$ en $g(x) = x + 1$. Berekende die volgende:

(4)

(1) x , if $\frac{1}{f(x) \cdot g(x)}$ is undefined.

(1) x , indien $\frac{1}{f(x) \cdot g(x)}$ ongedefinieerd is.

(4)

(2) x , if $f(x) \leq 0$

(2) x , indien $f(x) \leq 0$

(4)

(b) Consider the equation: $\sqrt{x+4} - 3 = x$

(b) Beskou die vergelyking: $\sqrt{x+4} - 3 = x$

(1) Show, without solving the equation, that $x \geq -4$.

(1) Toon sonder om die vergelyking op te los dat $x \geq -4$.

(2)

(2) Solve for x correct to one decimal place.

(2) Los op vir x korrek tot een desimale plek.

(6)

Question 4

Vraag 4

(a) Given: $f(x) = 2x^3$

(a) Gegewe: $f(x) = 2x^3$

(6)

(1) Determine the average gradient of f between the points $x = 1$ and $x = 1 + h$.

(1) Bepaal die gemiddelde gradient van f tussen die punte $x = 1$ en $x = 1 + h$.

(4)

(2) Hence, or otherwise, determine $f'(1)$.

(2) Bepaal vervolgens, of andersins, $f'(1)$.

(2)

(b) Determine $\frac{dy}{dx}$, $y = \frac{3}{x^2} - 10\sqrt{x}$.

(b) Bepaal $\frac{dy}{dx}$, $y = \frac{3}{x^2} - 10\sqrt{x}$.

[10]

Question 5

(a) Ryan opened a bank account 15 years ago, with the intention of saving money for when he retires.

The bank offered him an interest rate of 16% per annum compounded monthly for the first 5 years and thereafter changed the interest rate to 11% per annum (compounded annually).

Ryan made an immediate deposit of R300 000 upon opening the account, and withdrew R500 000 at the end of 13 years.

Calculate how much money he would have in this account at the end of the 15th year.

(b) If instead, Ryan had taken a retirement annuity over the same period of 15 years, and the insurance company had offered him 8% per annum compounded monthly, what would his monthly payments have been if he were to save an amount of R1 270 000 at the end of the 15th year.

Vraag 5

(a) Ryan het 15 jaar gelede 'n bankrekening geopen met die bedoeling om geld te spaar vir sy aftrede.

Die bank het hom 'n reënkoers van 16% per jaar maandeliks saamgestel vir die eerste 5 jaar aangebied en daarna die reënkoers verander na 11% per jaar (jaarliks saamgestel).

Ryan het 'n onmiddellike deposito van R300 000 gedoen toe hy die rekening geopen het en na alhoop van 13 jaar R500 000 onttrek.

Bereken hoeveel geld hy aan die einde van die 15^{de} jaar in hierdie rekening sou hê.

(b) Indien Ryan eerder 'n aftree-annuliet oor dieselfde tydperk van 15 jaar uitgeneem het en die verskeringsmaatskappy het hom 8% per jaar maandeliks saamgestel aangebied, wat sou sy maandelikse betalings gewees het indien hy 'n bedrag van R1 270 000 aan die einde van die 15^{de} jaar sou gespaar het?

The graph of f has a vertical asymptote at $x = -1$, both graphs intersect on the y -axis and the graph of g intersects the horizontal asymptote of f at the point $(-1; Y)$.

Die grafiek van f het 'n vertikale asimptoot by $x = -1$, die grafieke sny op die y -as en die grafiek van g sny die horisontale asimptoot van f by die punt $(-1; Y)$.

(a) Determine a , b and c . Show all working.

(a) Bepaal a , b en c . Toon alle berekeninge.

(b) If $f(x) = \frac{2}{x+1} + 3$ and $g(x) = 2x + 5$:

(b) Indien $f(x) = \frac{2}{x+1} + 3$ en $g(x) = 2x + 5$:

(1) Determine the x -intercepts of f and g .

(1) Bepaal die x -afsnitte van f en g .

(2) Hence, or otherwise, solve for x if $f(x) \cdot g(x) \leq 0$.

(2) Los verovloegs, of andersins, op vir x indien $f(x) \cdot g(x) \leq 0$.

(c) (1) Determine g^{-1} , the inverse of g in the form $y = \dots$

(c) (1) Bepaal g^{-1} , die inverse van g , in die vorm $y = \dots$

(2) Hence, or otherwise, determine the values of x for which $g^{-1}(x) > g(x)$.

(2) Bepaal verovloegs, of andersins, die waardes van x waarvoor $g^{-1}(x) > g(x)$.

SECTION B

Question 7

Answers only will not be awarded full marks.

AFDELING B

Vraag 7

Antwoorde alleen sal nie volpunte verdien nie.

(a) The roots of a quadratic equation are given as $5 - \sqrt{2}$ and $5 + \sqrt{2}$. Determine the equation in the form $ax^2 + bx + c = 0$.

(a) Die wortels van 'n kwadratiese vergelyking word gegee as $5 - \sqrt{2}$ en $5 + \sqrt{2}$. Bepaal die vergelyking in die vorm $ax^2 + bx + c = 0$.

(b) The equations $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ both have real and equal roots. Solve for a and b , where $a > 0$ and $b > 0$.

(b) Die vergelyking $x^2 + ax + b = 0$ en $x^2 + bx + a = 0$ het albei reële en gelyke wortels. Los op vir a en b , waar $a > 0$ en $b > 0$.

Question 8

Katy invested in Bitcoin (a digital currency) which increased in value at a rate of 200% per annum over a period of time.

Katy het in Bitcoin ('n digitale geldseenheid) belê wat teen 'n koers van 200% per jaar oor 'n tydperk in waarde toegeneem het.

Her original investment of (Y) rands was squared in value after (X) years when she sold her investment.

Haar oorspronklike belegging van Y rand het na X jaar in waarde gekwadrateer toe sy haar belegging verkoop het.

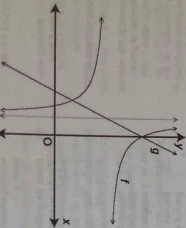
Question 6

In the diagram below, the graphs of

$$f(x) = \frac{a}{x} + c \text{ and } g(x) = 2x + 5 \text{ are given.}$$

In die diagram hieronder word die grafieke van

$$f(x) = \frac{a}{x} + c \text{ en } g(x) = 2x + 5 \text{ gegee.}$$



Memo: 134; 135

Memo: 135 - 137

- (a) Write down an equation representing the relationship between y and x in the form $y = \dots$
- (b) Sketch the graph of (a) showing any intercepts and asymptotes if they exist.
- (c) If Katy's original investment was R7500:
- Determine correct to the nearest month, the number of years it took to square in value.
 - Determine a restriction on the domain of the graph sketched in (b) that could represent Katy's investment.

Question 9

Consider the graphs of $g(x) = x^3 - 3x^2$ and $h(x) = -\frac{2}{3}x - \frac{4}{3}$.

- Determine whether the graph of h intersects the graph of g at its point of inflection. Show all working.
 - Determine the stationary point(s) of $y = g'(x)$. Classify your stationary point(s).
 - Hence, or otherwise, determine:
 - the value(s) of x for which g is concave down.
 - the gradient of the tangent to g at its point of inflection.
 - A student claims that the gradient of g at any point will never be less than -3 . Is the student correct? Explain.
- (c) Determine the value of k , if the graph of g is shifted so that the values of x for which the new graph $f(x) = (x + k)^3 - 3(x + k)^2$ decreases, is between -3 and -1 .

Vraag 9

Beskou die grafieke van $g(x) = x^3 - 3x^2$ en $h(x) = -\frac{2}{3}x - \frac{4}{3}$.

- Bepaal of die grafiek van h die grafiek van g by sy buigpunt sny. Toon alle berekeninge.
 - Bepaal die stasioneêre punt(e) van $y = g'(x)$. Klassifiseer jou stasioneêre punt(e).
 - Bepaal vervolgens, of andersins:
 - die waarde(s) van x waarvoor g konkav dawaarts is.
 - die gradient van die raaklyn aan g by sy buigpunt.
 - 'n Student beweer dat die gradient van g by enige punt nooit minder as -3 sal wees nie. Is die student korrek? Verduidelik.
- (c) Bepaal die waarde van k indien die grafiek van g geskuif word sodat die waardes van x waarvoor die nuwe grafiek $f(x) = (x + k)^3 - 3(x + k)^2$ afneem, tussen -3 en -1 is.

[10]

[11]

- Question 10**
- Given: $f(x) = ax^2 + bx + c$ where $b > 2a > 0$ and $a > c > 0$.
 - Show that $b^2 > 4ac$.
 - Draw a sketch graph of f .
 - Given: $g(x) = \frac{1}{x+2} - 1$ and $h(x) = 2^x + p$.
 - Sketch the graph of g .
 - Determine the value(s) of p for which $g(x) = h(x)$ has only one root.
- Vraag 10**
- Gegee: $f(x) = ax^2 + bx + c$ waar $b > 2a > 0$ en $a > c > 0$.
 - Toon dat $b^2 > 4ac$.
 - Teken 'n sketsgrafiek van f .
 - Gegee: $g(x) = \frac{1}{x+2} - 1$ en $h(x) = 2^x + p$.
 - Skets die grafiek van g .
 - Bepaal die waarde(s) van p waarvoor $g(x) = h(x)$ slegs een wortel het.

[11]

[12]

Question 11

Consider the word: CIRCLE

Note: The repeated letters are treated as identical.

- If two letters are selected at random, without replacement, determine the probability that:
 - Both letters are 'C'.
 - Only one letter is 'C'.
- Determine the number of different 6-letter arrangements that can be made with the letters.
 - Indien twee letters ewekansig gekies word sonder veranging, bepaal die waarskynlikheid dat:
 - Albei letters 'C' is.
 - Slegs een letter 'C' is.
 - Bepaal die getal verskillende 6-letter-rangskikkings wat met die letters gemaak kan word.
 - Hoewel woordrangskikkings kan gemaak word indien die woord met dieselfde letter begin en eindig?

Question 12

Lulu and Rempie have been invited to observe a missile testing experiment.

Lulu en Rempie is genooi om 'n missieloek-eksperiment waar te neem.

Vraag 12



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The missile engineer informed them that the probability that a missile will hit its target is 0,9.

He then asks Lulu and Riemple to work out the minimum number of missiles that would need to be fired at the target to ensure a 0,97 chance of hitting the target.

Lulu calculated that at least 2 missiles needed to be fired at the target.

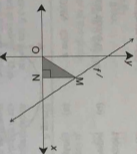
Riemple calculated that at least 3 missiles needed to be fired.

Determine who was correct. Show all calculations.

Question 13

In the diagram, the vertices of the shaded right-angled triangle OMN are O (0; 0), the variable point N (x_1 ; 0) which is on the x-axis where $0 \leq x_1 \leq 3$ and point M which lies on the line $2y + 3x - 6 = 0$.

The line represents the graph of the first derivative function of the function f .



The area of the shaded region is given as $A = rx^2 + tx$ and $f(x) = rx^2 + bx + c$ has a stationary point at (x ; 5).

Determine whether the value of x_1 that yields the maximum area of ΔOMN is also the value of x_2 that yields the maximum distance between the graphs of f and its derivative f' .

Show all working.

Die missielingenieur het hulle ingelig dat die waarskynlikheid dat 'n missiel sy teiken sal tref 0,9 is.

Hy vra toe vir Lulu en Riemple om uit te werk wat die minimum getal missiele is wat op die teiken afgewurp sal moet word om 'n 0,97-kans dat die teiken getref sal word, te verseker.

Lulu het bereken dat minstens 2 missiele op die teiken afgewurp moet word.

Riemple het bereken dat minstens 3 missiele afgewurp moet word.

Bepaal wie korrek was. Toon alle berekeninge. [6]

Vraag 13

In die diagram is die hoekpunte van die gearseerde reghoekige driehoek OMN punt O (0; 0), die veranderlike punt N (x_1 ; 0) wat op die x-aks is waar $0 \leq x_1 \leq 3$ en punt M wat op die lyn $2y + 3x - 6 = 0$ lê.

Die lyn verteenwoordig die grafiek van die eerste afgeleide-funksie van die funksie van f .

Die oppervlakte van die gearseerde gebied word gegee as $A = rx^2 + tx$ en $f(x) = rx^2 + bx + c$ het 'n stasionêre punt by (x ; 5).

Bepaal of die waarde van x_1 wat die maksimum oppervlakte van ΔOMN oplewer ook die waarde is van x_2 wat die maksimum afstand tussen die grafieke van f en sy afgeleide f' oplewer.

Toon alle berekeninge.