$\therefore 2DB = BE$

$DE^2 = DB^2 + BE^2$ — Pythagoras

$\therefore 6^2 = DB^2 + BE^2$

$\therefore 36 = DB^2 + BE^2$

$\therefore 36 = DB^2 + (2DB)^2$ — $2DB = BE$

$\therefore 36 = DB^2 + 4DB^2$

$\therefore 36 = 5DB^2$

$\therefore DB^2 = \frac{36}{5}$

$\therefore DB = \sqrt{\frac{36}{5}} = \frac{6}{\sqrt{5}} = 2{,}6832... \approx 2{,}68$ units/*eenhede*

ieb
Nov 2018
Paper 1 / Vraestel 1

QUESTION 1 / VRAAG 1

(a) $T_{100} = 512;\ d = 7$

$T_n = a + (n-1)d$

$\therefore 512 = a + (100-1)(7)$

$\therefore 512 = a + (99)(7)$

$\therefore 512 = a + 693$

$\therefore a = 512 - 693 = -181$

(b)(1) $T_n = 2n + 3$

$T_{n+1} - T_n = 2(n+1) + 3 - (2n+3)$

$= 2n + 2 + 3 - 2n - 3$

$= 2$

$\therefore$ first differences is a constant

$\therefore$ the sequence is arithmetic

$\therefore$ *die eerste verskille is 'n konstante*

$\therefore$ *dit is 'n rekenkundige ry*

(b)(2) $T_1 = 2(1) + 3 = 2 + 3 = 5$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}[2(5) + (n-1)(2)]$

$= \frac{n}{2}[10 + 2n - 2]$

$= \frac{n}{2}[8 + 2n]$

$= 4n + n^2$

(c)

4   7   14   25

  3   7   11

   4   4

$T_n = an^2 + bn + c$

$T_2 = a(2)^2 + b(2) + c \quad = 4a + 2b + c$

$T_1 = a(1)^2 + b(1) + c \quad = a + b + c$

$\therefore T_2 - T_1 = 3a + b$

$2a = 4$

$\therefore a = 2$

$T_2 - T_1 = 3a + b$

$\therefore 3 = 3(2) + b$

$\therefore b = 3 - 6$

$\therefore b = -3$

$a + b + c = T_1$

$2 - 3 + c = 4$

$\therefore c = 4 - 2 + 3$

$\therefore c = 5$

$T_n = 2n^2 - 3n + 5$

QUESTION 2 / VRAAG 2

(a)(1) $\sum_{n=1}^{x} 108 \times \left(\frac{2}{3}\right)^n$

$T_1 = 108 \times \left(\frac{2}{3}\right)^1 = 108 \times \left(\frac{2}{3}\right) = 72$

$T_2 = 108 \times \left(\frac{2}{3}\right)^2 = 108 \times \left(\frac{4}{9}\right) = 48$

(a)(2) $\sum_{n=1}^{x} 108 \times \left(\frac{2}{3}\right)^n = \frac{520}{3}$

$S_n = \frac{a(r^n - 1)}{r - 1}$

$\frac{520}{3} = \frac{72\left[\left(\frac{2}{3}\right)^x - 1\right]}{\frac{2}{3} - 1}$

$\frac{520}{3} = \frac{72\left[\left(\frac{2}{3}\right)^x - 1\right]}{-\frac{1}{3}}$

$\frac{520}{3} = -216\left[\left(\frac{2}{3}\right)^x - 1\right]$

$[\div(-216)]$: $\left[\left(\frac{2}{3}\right)^x - 1\right] = \frac{520}{3} \div (-216)$

$\left(\frac{2}{3}\right)^x - 1 = -\frac{65}{81}$

$\left(\frac{2}{3}\right)^x = 1 - \frac{65}{81}$

$\left(\frac{2}{3}\right)^x = \frac{16}{81}$

$\left(\frac{2}{3}\right)^x = \left(\frac{2}{3}\right)^4$

$\therefore x = 4$

(b) $2\pi(21)^2 + 2\pi(3)^2 + 2\pi\left(\frac{3}{7}\right)^2 + ...$

$\therefore a = 2\pi(21)^2$ **and / en**

$r = \frac{T_2}{T_1} = \frac{2\pi(3)^2}{2\pi(21)^2} = \left(\frac{3}{21}\right)^2 = \left(\frac{1}{7}\right)^2$

$S_\infty = \frac{a}{1-r} = \frac{2\pi(21)^2}{1-\left(\frac{1}{7}\right)^2} = \frac{2\pi(21)^2}{1-\frac{1}{49}} = \frac{2\pi(21)^2}{\frac{48}{49}}$

$= 2828{,}611485$

$\approx 2\ 828{,}61$

QUESTION 3 / VRAAG 3

(a)(1) undefined if: / *ongedefinieerd indien:*

$f(x) = 0$ or / of $g(x) = 0$

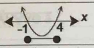
$\therefore x^2 - 3x - 4 = 0$ or / of $x + 1 = 0$

$\therefore (x+1)(x-4) = 0$ or / of $x = -1$

$\therefore [x = -1$ or / of $x = 4]$ or / of $x = -1$

$\therefore x = -1$ or / of $x = 4$

(a)(2) $f(x) = (x+1)(x-4)$



$\therefore (x+1)(x-4) \le 0$

$\therefore -1 \le x \le 4$

(b)(1) for a real solution: / *vir 'n reële oplossing:*

$x + 4 \ge 0$

$\therefore x \ge -4$

(b)(2) $\sqrt{x+4} - 3 = x$

$\sqrt{x+4} = x + 3$

$\left(\sqrt{x+4}\right)^2 = (x+3)^2$

$x + 4 = x^2 + 6x + 9$

$0 = x^2 + 5x + 5$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-5 \pm \sqrt{5^2 - 4(1)(5)}}{2(1)}$

$x = \frac{-5 \pm \sqrt{25 - 20}}{2}$

$x = \frac{-5 \pm \sqrt{5}}{2}$

$x = \frac{-5 \pm 2{,}236067977}{2}$

$\therefore x = -1{,}381966011$ or / of $x = -3{,}618033989$

$\therefore x \approx -1{,}4$ or / of $x = -3{,}6$ **---- na / nvt**

$\therefore x \approx -1{,}4$

QUESTION 4 / VRAAG 4

(a)(1) $f(x) = 2x^3$

$f(1) = 2(1)^3 = 2(1) = 2$

$f(1+h) = 2(1+h)^3$

$= 2(1+h)(1+h)(1+h)$

$= 2(1 + 2h + h^2)(1+h)$

$= 2(1 + 2h + h^2 + h + 2h^2 + h^3)$

$= 2(1 + 3h + 3h^2 + h^3)$

$= 2 + 6h + 6h^2 + 2h^3$

average gradient / *gemiddelde helling*

$= \frac{f(1+h) - f(1)}{(1+h) - 1}$

$= \frac{2 + 6h + 6h^2 + 2h^3 - 2}{1 + h - 1}$

$= \frac{6h + 6h^2 + 2h^3}{h}$

$= \frac{h(6 + 6h + 2h^2)}{h}$

$= 6 + 6h + 2h^2$

(a)(2) $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$

$= \lim_{h \to 0}(6 + 6h + 2h^2)$ from / *uit* **(a)(1)**

$= 6$

(b) $y = \frac{3}{x^2} - 10\sqrt[5]{x}$

$\therefore y = 3x^{-2} - 10.x^{\frac{1}{5}}$

$\frac{dy}{dx} = -6x^{-3} - 2x^{-\frac{4}{5}} = -\frac{6}{x^3} - \frac{2}{\sqrt[5]{x^4}}$

QUESTION 5 / VRAAG 5

(a)

R300 000		-R500 000	
T_0	T_5	T_{13}	T_{15}
$i = \frac{0{,}16}{12}$, n = 60 mnths/ *mnde*	i = 0,11, n = 8 years/ *jaar*	i = 0,11, n = 2 years/ *jaar*	

At T_5 / By T_5: $A = P(1+i)^n$

$A = 300\ 000\left(1 + \frac{0{,}16}{12}\right)^{60} = R664\ 142{,}0648$

At T_{13} / By T_{13}:

$A = P(1+i)^n$

$A = 664\ 142{,}0648(1 + 0{,}11)^8 = R1\ 530\ 540{,}473$

$P = R1\ 530\ 540{,}473 - R500\ 000 = R1\ 030\ 540{,}473$

At T_{15} / By T_{15}:

$A = P(1+i)^n$

$A = 1\ 030\ 540{,}473(1 + 0{,}11)^2 = R1\ 269\ 728{,}917$

(b) $F = \frac{x[(1+i)^n - 1]}{i}$

$$1\,270\,000 = \frac{x\left[\left(1+\frac{0{,}08}{12}\right)^{180}-1\right]}{\frac{0{,}08}{12}}$$

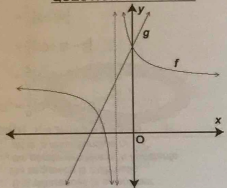
$1\,270\,000 = x(346{,}0382216)$

$\therefore x = \dfrac{1\,270\,000}{346{,}0382216}$

$= R3\,670{,}114804$

$\approx R3\,670{,}11$

QUESTION 6/ VRAAG 6



(a) $g(x) = 2x+5$

y-intercept / *y-afsnit* : (0; 5)

g intersect the horizontal asymptote at $(-1; y)$

g sny die horisontale asimptoot by $(-1; y)$

$g(-1) = 2(-1)+5 = -2+5 = 3$

$\therefore\ (-1;3)$

$\therefore$ horizontal asymptote / *horisontale asimptoot*

$y = 3 \qquad \therefore c = 3$

$f(x) = \dfrac{a}{x+b}+3$

asymptote at $x=-1$ / *asimptoot by* $x=-1$

$\therefore b = 1$

$\therefore f(x) = \dfrac{a}{x+1}+3$

(0; 5): $5 = \dfrac{a}{0+1}+3$

$5 = \dfrac{a}{1}+3 \qquad \therefore 5 = a+3 \qquad \therefore a = 5-3 = 2$

$\therefore f(x) = \dfrac{2}{x+1}+3$

(b)(1) x-intercept of f / *x-afsnit van f* : $f(x) = 0$

$\therefore 0 = \dfrac{2}{x+1}+3$

$\therefore -3 = \dfrac{2}{x+1}$

$\therefore -3(x+1) = 2$

$\therefore -3x-3 = 2$

$\therefore -3x = 5$

$\therefore x = -\dfrac{5}{3}$

x-intercept of g / *x-afsnit van g* : $g(x) = 0$

$\therefore\ 0 = 2x+5 \quad \therefore x = -\dfrac{5}{2}$

(b)(2) $x \le -\dfrac{5}{2}$ or / of $-\dfrac{5}{3} \le x < -1$

(c)(1) inverse: $x = 2y+5$

$\therefore x-5 = 2y$

[÷2] $\quad \therefore y = \dfrac{x-5}{2} = \dfrac{1}{2}x - \dfrac{5}{2}$

(c)(2) $\dfrac{1}{2}x - \dfrac{5}{2} = 2x+5$

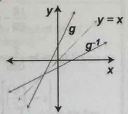
[x2]: $\quad x-5 = 4x+10$

$-3x = 15$

[÷(-3)] $\quad x = -5$

$\therefore g^{-1}(x) > g(x)$

if / *as* $\quad x < -5$



QUESTION 7/ VRAAG 7

(a) $(x-x_1)(x-x_2) = 0$

$\left(x-(5-\sqrt{2})\right)\left(x-(5+\sqrt{2})\right) = 0$

$\left((x-5)+\sqrt{2}\right)\left((x-5)-\sqrt{2}\right) = 0$

$(x-5)^2 - \left(\sqrt{2}\right)^2 = 0$

$(x-5)^2 - 2 = 0$

$x^2-10x+25-2 = 0$

$x^2-10x+23 = 0$

(b) $x^2+ax+b = 0 \qquad | \qquad x^2+bx+a = 0$

$\Delta = b^2-4ac \qquad | \qquad \Delta = b^2-4ac$

For real and equal roots: $\Delta = 0$

Vir reële en gelyke wortels: $\Delta = 0$

$\therefore 0 = a^2-4(1)(b) \qquad | \qquad \therefore 0 = b^2-4(1)(a)$

$0 = a^2-4b \qquad | \qquad 0 = b^2-4a$ ----(2)

$4b = a^2$

$\therefore b = \dfrac{a^2}{4}$ ----(1)

(1) in (2): $0 = \left(\dfrac{a^2}{4}\right)^2 - 4a$

$0 = \dfrac{a^4}{16} - 4a$

[x16] $\quad 0 = a^4-64a$

$0 = a(a^3-64)$

$\therefore a = 0$ or / of $a^3 = 64$

but / *maar* $a > 0 \qquad a^3 = 4^3$

$\therefore a = 4$ -----(3)

(3) in (1): $\therefore b = \dfrac{4^2}{4} = 4$

QUESTION 8/ VRAAG 8

(a) $r = 200\%$

$\therefore i = \dfrac{200}{100} = 2$

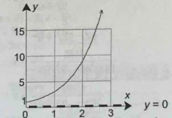
$A = P(1+i)^n$

$y^2 = y(1+2)^x$

[÷y] $\quad y = (1+2)^x$

$\therefore y = 3^x$

(b)



asymptote at $y=0$ / *asimptoot by* $y=0$

(c)(1) $y = 3^x$

$750 = 3^x$

$\therefore x = \log_3 750 = 6{,}025850...$

$\therefore x = 6$ years and / *jaar en*

$(0{,}025850316..\times 12)$ months / *maande*

$\therefore x = 6$ years and / *jaar en*

0,310... months / *maande*

$\therefore x \approx 6$ years / *jaar*

(c)(2) $x > 6$

QUESTION 9/ VRAAG 9

(a) $g(x) = x^3-3x^2$

$g'(x) = 3x^2-6x$

$g''(x) = 6x-6$

point of inflection / ***buigpunt*** **:** $g''(x) = 0$

$\therefore 6x-6 = 0 \qquad \therefore x = 1$

$g(1) = (1)^3-3(1)^2 = 1-3 = -2$

$\therefore (1;-2)$

$h(x) = -\dfrac{2}{3}x - \dfrac{4}{3}$

$h(1) = -\dfrac{2}{3}(1) - \dfrac{4}{3} = -\dfrac{2}{3} - \dfrac{4}{3} = -\dfrac{6}{3} = -2$

The graph of h does intersect the graph of g at its point of inflection.

Die grafiek van h sny die grafiek van g by sy buigpunt.

(b)(1) $y = g'(x) = 3x^2-6x$

Stationary points if / *Stasionêre punte indien:*

$\dfrac{dy}{dx} = 0 \qquad \therefore 6x-6 = 0$

$\therefore x = 1$

$x = 1: \quad y = 3(1)^2-6(1) = 3-6 = -3 \quad \therefore (1;-3)$

$D_x[6x-6] = 6 > 0$

$\therefore (1;-3)$ is a minimum turning point.

$\therefore (1;-3)$ *is 'n minimum draaipunt.*

(b)(2)(i) $g(x) = x^3-3x^2$

$g''(x) = 6x-6$

concave downward if:/ *konkaaf afwaarts indien:*

$g''(x) < 0$

$\therefore 6x-6 < 0$

$\therefore x < 1$

(b)(2)(ii) point of inflection of g: / *buigpunt van g:*

$g''(x) = 6x-6 = 0$

$\therefore x = 1 \qquad$ [from / *uit* (b)(1)]

$m = g'(x) = 3x^2-6x$

$x = 1: \ g'(1) = 3(1)^2-6(1) = 3-6 = -3$

$\therefore$ gradient at point of inflection is -3 /

helling by die buigpunt is -3

(b)(3) $m = g'(x) = 3x^2-6x$

$= 3(x^2-2x)$

$= 3(x^2-2x+1)-3$

$= 3(x-1)^2-3$

Or from **(b)(1)** *Of uit* **(b)(1)**

$\therefore\ g'(x)$ has a minimum value of -3, the student is correct.

$g'(x)$ *het 'n minimum-waarde van* -3*, die student is korrek*

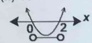
(c) g decreases if $m = g'(x) < 0$

g neem af indien $m = g'(x) < 0$

$\therefore 3x^2-6x < 0$

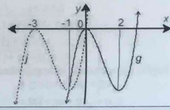
$\therefore 3x(x-2) < 0$

$\therefore 0 < x < 2$



j must decrease between -3 and -1 /

j moet afneem tussen -3 en -1



$j(x) = g(x+k) = (x+k)^3 - 3(x+k)^2$

$\therefore$ *g* move 3 units to the left to form *j* /
g skuif 3 eenhede na links om j te vorm

$\therefore k = 3$

QUESTION 10/ VRAAG 10

(a)(1) $b > 2a$

$\therefore b^2 > (2a)^2$ [b > 0 and / *en* 2a > 0]

$\therefore b^2 > 4a^2$

$a > c > 0$

$\therefore b^2 > 4a \times a > 4a \times c$

$\therefore b^2 > 4ac$

(a)(2) $\therefore b^2 > 4ac$ from / *uit* **(a)(1)**

$\therefore b^2 - 4ac > 0$

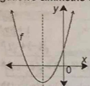
$\therefore$ roots are unequal and real
wortels is ongelyk en reëel

$a > 0$ $\therefore$ concave upwards / *konkaaf opwaarts*

$c > 0$ $\therefore$ y-intrecept >0 / *y-afsnit > 0*

$b > 0;\ a > 0 \quad \therefore \frac{-b}{2a} < 0$

$\therefore$ negative axis of symmetry /
$\therefore$ *negatiewe simmetrie-as*



(b)(1) $g(x) = \frac{1}{x+2} - \frac{1}{2}$

y-intercept / *y-afsnit* : x = 0

$y = \frac{1}{0+2} - \frac{1}{2} = \frac{1}{2} - \frac{1}{2} = 0 \quad \therefore (0;0)$

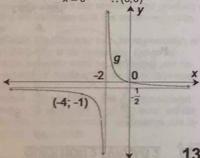
x-intercept / *x-afsnit*: y = 0

$0 = \frac{1}{x+2} - \frac{1}{2}$

[x 2(x + 2)] $\quad 0 = 2 - (x+2)$

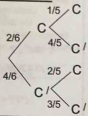
$0 = 2 - x - 2$

$x = 0 \quad \therefore (0;0)$



(b)(2) $p \geq -\frac{1}{2}$

QUESTION 11/ VRAAG 11



(a)(1) $\frac{2}{6} \times \frac{1}{5} = \frac{1}{15}$

(a)(2) $\frac{2}{6} \times \frac{4}{5} + \frac{4}{6} \times \frac{2}{5}$

$= \frac{4}{15} + \frac{4}{15}$

$= \frac{8}{15}$

(b) $\frac{6!}{2!} = \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 1} = 360$

(c) starts and ends with a C /
Begin en eindig met 'n C

1 x (4 x 3 x 2 x 1) x 1 = 24

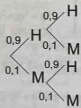
QUESTION 12/ VRAAG 12

H: hit / *tref* **M:** miss / *mis*

One missile / *een missiel*

P(H) = 0,9

Two missiles / *twee missiele*



HH $0{,}9 \times 0{,}9 = 0{,}81$

HM $0{,}9 \times 0{,}1 = 0{,}09$

MH $0{,}1 \times 0{,}9 = 0{,}09$

MM $0{,}1 \times 0{,}1 = 0{,}01$

P(H) = 0,81 + 0,09 + 0,09 = 0,99 > 0,97

$\therefore$ at least 2 missiles / *ten minste 2 missiele*

$\therefore$ Lulu is correct / *Lulu is korrek*

QUESTION 13/ VRAAG 13

$2y + 3x - 6 = 0$

$2y = -3x + 6$

[÷2] $\quad \therefore y = -\frac{3}{2}x + 3$

$\therefore M\left(x; -\frac{3}{2}x + 3\right)$

A = Area $\triangle$ OMN = $\frac{1}{2}$ base x $\perp$ height /
= $\frac{1}{2}$ *basis* x $\perp$ *hoogte*

$A = \frac{1}{2}.x.\left(-\frac{3}{2}x + 3\right)$

$= -\frac{3}{4}x^2 + \frac{3}{2}x$

For maximum area: / *Vir maksimum oppervlakte:*

$\frac{dA}{dx} = 0$

$\therefore -\frac{3}{2}x + \frac{3}{2} = 0$

$\therefore \frac{3}{2}x = \frac{3}{2}$

$\therefore x_1 = 1$

$f(x) = rx^2 + bx + c$

$f'(x) = 2rx + b$ and / *en* $\quad y = -\frac{3}{2}x + 3$

$\therefore 2r = -\frac{3}{2}$ and / *en* $b = 3$

$[\times \frac{1}{2}]: \ r = -\frac{3}{4}$

$\therefore f(x) = -\frac{3}{4}x^2 + 3x + c$

and / *en* $f'(x) = 2\left(-\frac{3}{4}\right)x + 3$

$\therefore f'(x) = -\frac{3}{2}x + 3$

$\therefore f(x) - f'(x) = -\frac{3}{4}x^2 + 3x + c - \left(-\frac{3}{2}x + 3\right)$

$\therefore f(x) - f'(x) = -\frac{3}{4}x^2 + 3x + c + \frac{3}{2}x - 3$

$= -\frac{3}{4}x^2 + \frac{9}{2}x + c - 3$

For a maximum distance /
Vir 'n maksimum afstand

$D_x\left[-\frac{3}{4}x^2 + \frac{9}{2}x + c - 3\right] = 0$

$\therefore -\frac{3}{2}x + \frac{9}{2} = 0$

$\therefore \frac{3}{2}x = \frac{9}{2}$

$\left[\times \frac{2}{3}\right] \quad x = \frac{9}{2} \times \frac{2}{3} = 3$

$\therefore x_2 = 3$

$\therefore x_1 \neq x_2$

ieb
Nov 2018
Paper 2 / Vraestel 2

QUESTION 1/ VRAAG 1

(a)