## MATHEMATICS: PAPER II

## EXAMINATION NUMBER

$\square$
Time: 3 hours
150 marks

## PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 24 pages and an Information Sheet of 2 pages (i-ii). Please check that your question paper is complete.
2. Read the questions carefully.
3. Answer ALL the questions on the question paper and hand it in at the end of the examination. Remember to write your examination number in the space provided.
4. Diagrams are not necessarily drawn to scale.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. Ensure that your calculator is in DEGREE mode.
7. All the necessary working details must be clearly shown. Answers only will not necessarily be awarded full marks.
8. It is in your own interest to write legibly and to present your work neatly.
9. Round off to one decimal place unless otherwise stated.

FOR OFFICE USE ONLY: MARKER TO ENTER MARKS

| Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 | Q8 | Q9 | Q10 | Q11 | Q12 | TOTAL |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
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| 17 | 10 | 10 | 23 | 9 | 9 | 8 | 19 | 9 | 10 | 13 | 13 | 1150 |

## SECTION A

## QUESTION 1

In the diagram below M is the midpoint of the line that joins $\mathrm{A}(1 ;-1)$ and $\mathrm{B}(5 ; 3)$.

(a) Find the coordinates of point $M$, the midpoint of line $A B$.
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(b) Find the equation of a line perpendicular to $A B$ that goes through $M$ (the perpendicular bisector of line $A B$ ).
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$\qquad$
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$\qquad$ (4)
(c) Calculate the length of line $A B$. (Leave your answer in surd form.)
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$\qquad$
(d) Determine the equation of a circle that has a centre at M and goes through points $A$ and $B$ in the form of $x^{2}+y^{2}+a x+b y+c=0$.
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(e) Determine the value(s) of $k$, if point $\mathrm{H}(4 ; k)$ lies on the circle.
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(f) Determine the shortest distance of the circle from the $y$-axis.
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## QUESTION 2

In the diagram below, circle centre O is drawn in the Cartesian plane.

- MN is a tangent to the circle at M .
- $\quad \mathrm{N}$ is a point outside the circle with co-ordinates $\mathrm{N}(11 ;-5)$.

(a) Write down the size of $O \hat{M} N$. Give a reason for your answer.
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$\qquad$
$\qquad$
(b) If the equation of the circle above is $x^{2}-8 x+(y+4)^{2}=9$ then:
(1) Determine the coordinates of O .
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$\qquad$
(2) Determine the length of OM .
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(c) Calculate the length of MN.
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## QUESTION 3

On the set of axis below the graph of $f(x)=\cos 2 x$ with $x \in\left[0^{\circ} ; 180^{\circ}\right]$ has been sketched.

(a) On the set of axis shown above sketch the graph of $g(x)=3 \sin 2 x$ with $x \in\left[0^{\circ} ; 180^{\circ}\right]$.
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$\qquad$ (4)
(b) Calculate the values for $x$ where $f(x)=g(x)$ if $x \in\left[0^{\circ} ; 180^{\circ}\right]$.
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(c) For what values of $x$ will $\frac{g(x)}{f(x)}$ be undefined if $x \in\left[0^{\circ} ; 180^{\circ}\right]$ ?
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## QUESTION 4

(a) Use the diagram below to prove the theorem that states:
"The angle subtended by a chord at the centre of a circle is twice the size of the angle that it subtends at the circle."


Required to prove: $A \hat{O} C=2 \times A \hat{B} C$
Construction:
Proof: $\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$ (5)
(b) In the diagram below, circle centre O is drawn.

- $S, T, B$ and $M$ are points on the circle.
- Line MT cuts line OB at point $E$.
- $M \hat{S} T=67^{\circ}$.
- $O \hat{B} M=40^{\circ}$.

(1) Find the size of $\hat{B}_{1}$.
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$\qquad$
(2) Determine the size of $\hat{T}_{2}$.
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$\qquad$
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$\qquad$
$\qquad$ (4)
(3) Find the size of $\hat{M}_{2}$.
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$\qquad$
(c) In the diagram below, P, Q, R and S are points on the circle. QS and PR intersect at point $T$.
- The line from V is a tangent at P .
- $Q \hat{R} P=56^{\circ}$.
- $R \hat{P} V=70^{\circ}$.

(1) Find the size of $R \hat{S} T$.
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$\qquad$
$\qquad$
$\qquad$
(2) If $\hat{Q}_{1}=37^{\circ}$, then explain why QS is not the diameter of the circle.
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## QUESTION 5

In the diagram below, $A, B, C$ and $E$ are points on a circle. $A C$ and $B E$ intersect at point $F$. $D$ is a point outside the circle. CD and DE are drawn.

- $A \hat{B} E=x$.
- $B \hat{E} C=y$.
- $C \hat{D} E=\theta$.

(a) If $\theta=x+y$ then prove that FCDE is a cyclic quadrilateral.
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(b) If $A B=A E$, then prove that line $A E$ is a tangent to the circle that goes through $F, C, D$ and $E$.
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$\qquad$ (4)
[9]


## QUESTION 6

Below is a cumulative frequency graph showing the average number of litres of water used per household per day in a town.


Other information given:

- The lowest amount used by any household during a day is 20 litres of water.
- The highest amount used by any household during a day is 280 litres of water.
(a) Sketch a box and whisker plot to represent the average number of litres used per household per day.

(b) How many households use more than 100 litres of water per day?
$\qquad$
(c) Describe the skewness of the data. (Give a reason for your answer.)
$\qquad$
$\qquad$
(d) As a result of a price increase, many of the households using in excess of 100 litres of water per day on average intend to reduce their water consumption substantially. How would this affect
(1) standard deviation? (Explain your answer.)
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(2) the skewness of the data? (Explain your answer.)
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## SECTION B

## QUESTION 7

Information about advertising via pamphlets and television separately is given in the table below.

- The correlation between the expenditure (in rand) on advertising via pamphlets ( $x$ ) and sales ( $y$ ) is recorded in the first row.
- The correlation between expenditure (in rand) on television advertising ( $x$ ) and sales $(y)$ is recorded in the second row of the table.

|  | Line of best fit <br> (including outliers) | Correlation <br> Coefficient | The number of <br> outliers with high <br> expenditure and <br> LOW sales | The number of <br> outliers with low <br> expenditure and <br> HIGH sales |
| :---: | :---: | :---: | :---: | :---: |
| Pamphlets | $\mathrm{y}=11000+2 x$ | 0,95 | 1 | 0 |
| Television | $\mathrm{y}=8000+4 x$ | 0,88 | 0 | 3 |

(a) Which line of best fit would be the most accurate? (Give a reason.)
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$\qquad$
(b) If the outliers were removed, how would this affect the:
(1) correlation coefficient of television advertising? (Give a reason.)
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$\qquad$
(2) gradient for each line of best fit? (Give a reason.)
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(c) Using the information in the table above, explain which method of advertising you would use and why.
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## QUESTION 8

(a) If $\sin 64^{\circ}=p$, determine the value of the following in terms of $p$ :
(1) $\cos 334^{\circ} \cdot \sin 244^{\circ}$
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$\qquad$
$\qquad$
(2) $8 \sin 16^{\circ} \cdot \cos 16^{\circ} \cdot \cos 32^{\circ}$
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$\qquad$
(b) If $\sin 43^{\circ}=A$ and $\cos \left(90^{\circ}-k\right) \cos 23^{\circ}+\cos 246^{\circ} \sin 23^{\circ}=B$, then find the value of $k$ if $\mathrm{A}=\mathrm{B}$ and $0^{\circ} \leq k \leq 90^{\circ}$.
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(c) Prove that: $\frac{2 \cos 2 \theta \cdot \cos \theta}{\cos ^{2} \theta-\sin ^{2} \theta}+2 \tan \theta \cdot \sin \theta=\frac{2}{\cos \theta}$.
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(d) In the diagram below, $\Delta$ FGW is drawn.

- $\mathrm{FW}=6$ units.
- $F G=\sqrt{116}$ units.
- $G \hat{W} H=45^{\circ}$.


Calculate the length of GH.
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## QUESTION 9

In the diagram below, $\triangle \mathrm{AFC}$ is drawn. B lies on AC with D and E on CF .

- BD//AE and BE//AF.
- $\mathrm{CB}: С А=5: 9$.

(a) If the length of $C A$ is 18 units, find the length of BA.
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$\qquad$
(b) Determine the $\frac{\text { Area of } \triangle B D C}{\text { Area of } \triangle B E D}$.
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(c) If the length of CF is 45 units, find the length of $D E$.
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## QUESTION 10

In the diagram below, circle centre O passes through F, S, H and G.

- DF and $D S$ are tangents to the circle at $F$ and $S$ respectively.
- DF/ISG.
- FOH intersects SG at R.

(a) Prove that $\triangle D S F / / / \triangle O H G$.
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(b) Show that $2 \times D F=\frac{S F \times F H}{H G}$.
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## QUESTION 11

In the diagram below, the given shape is enclosed by line $A B$, line $B C$ and part of a circle with centre $O$.

- $A B=\sqrt{13}$ units.
- $B C=\sqrt{29}$ units.
- $A C=8$ units.
- $A, K$ and $C$ are points on the circle.
- EK is perpendicular to AC and goes through O.

(a) If $\mathrm{OE}=0,8$ units, find the length of EK .
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(b) You cut out the shape above and fold it along the dotted line AC so that $\triangle A B C$ is on the vertical plane and the circular shape through the points $A, K$ and $C$ is on the horizontal plane.

Calculate the straight line distance from $B$ to $K$ once it has been folded.
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## QUESTION 12

A farmer uses a drone to check his fences for any damage.

- The drone flies in a perfect circle and passes directly above points $A, B$ and $C$.
- Points $A, B, C$ and $D$ are on the same horizontal plane.
- D lies on the $x$-axis.
- At present the angle made between line AD and the $x$-axis is $45^{\circ}$.

(a) Determine the current coordinates of point D.
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(b) Calculate the size of $A \hat{B} C$.
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(c) How far to the right must $D$ be moved along the $x$-axis so that the drone will fly directly above it on its circular flight path?
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